A114757

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: NRL Memorandum Report 4556
2. GOVT ACCESSION NO.:
3. RECIPIENT'S CATALOG NUMBER:
4. TITLE (and Subtitle): ENERGY LOSSES FROM A HEATED AIR COLUMN
5. TYPE OF REPORT & PERIOD COVERED: Interim report on a continuing problem
6. PERFORMING ORG. REPORT NUMBER:
7. AUTHOR(s): M. Raleigh, J. R. Greig, and R. F. Fernsler*
8. CONTRACT OR GRANT NUMBER(s):
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Naval Research Laboratory, Washington, D.C. 20375
10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: 61153N; R011-09-41; 67-0871-0-1; and 61101E; 0; OR40AA
11. CONTROLLING OFFICE NAME AND ADDRESS: Office of Naval Research, Arlington, VA 22217; Defense Advanced Research Projects Agency, Arlington, VA 22209 ATTN: Program Management/MIS
12. REPORT DATE: May 17, 1982
13. NUMBER OF PAGES: 17
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office): Naval Surface Weapons Center, White Oak, MD 20910 ATTN: R-401
15. SECURITY CLASS. (of this report):
15a. DECLASSIFICATION/DOWNGRADING SCHEDULE:
16. DISTRIBUTION STATEMENT (of this Report): Approved for public release; distribution unlimited.
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report):
18. SUPPLEMENTARY NOTES:

Research supported by the Office of Naval Research and by the Defense Advanced Research Projects Agency (DoD) ARPA Order No. 3718, monitored by the Naval Surface Weapons Center under Contract N60921-81-WR-W0114.

*Permanent address: JAYCOR, Alexandria, VA 22304

19. KEY WORDS (Continue on reverse side if necessary and identify by block number):
Reduced density channel
Energy loss
Radiation
Heat conduction

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Estimates are made of the energy losses from the hot channel produced by (laser-guided) electric discharges in the atmosphere. The loss processes considered are heat conduction, continuum radiation, and line radiation. For the conditions calculated to exist in such channels ($T_e \approx 20{,}000$ K, $\Sigma N_i = 2 \times 10^{18}$ cm^{-3}), the total energy loss is found to be small compared to the measured energy input (~3 J/cm along the length of the channel). Therefore the assumption of negligible losses used to calculate the channel conditions is valid and the expansion of the hot channel will be adiabatic except where compressional shock waves are involved.

(Continued)

DD FORM 1473, 1 JAN 73 EDITION OF 1 NOV 65 IS OBSOLETE
S/N 0102-014-6601

20. ABSTRACT (Continued)

Since the dominant energy loss is through optically thick line radiation and this loss is strongly temperature dependent, such radiation effectively limits the temperature to which air can be heated to $\lesssim$ 20,000 K.

CONTENTS

ENERGY LOSSES FROM A HEATED AIR COLUMN

I. INTRODUCTION

Experiments at NRL which use the reduced-density channels produced by laser-guided, electric discharges in the atmosphere have been described.[1] Initially these channels are narrow, hot, at full atmospheric density, and highly overpressured. They expand in $\lesssim 30\ \mu s$ to form atmospheric pressure, quasi-stable, reduced density channels. Simulations of the ohmic heating process using an air chemistry code[2] have assumed radiation transfer and shock heating may be neglected. Hydrocode simulations of the evolution of the channel density profile have also assumed the expansion process taking place behind the outgoing shock wave is adiabatic.[3]

We estimate the radiation loss rate from the channel at peak current using the air chemistry code predictions of the channel conditions. We also estimate absorption lengths both inside and outside the channel. The radiation loss mechanisms which have been considered are continuum emission due to electron-ion bremsstrahlung, continuum emission due to electron-neutral bremsstrahlung, continuum emission due to free-bound transitions, and line radiation. Heat conduction losses are estimated for 30 μs, i.e., during the whole expansion phase, and proven negligible. These estimates provide reasonable support for the assumptions made in the codes.

II. CHANNEL CONDITIONS

The experimental current and voltage signals for the electric discharge which heats the channels were simulated by combining the CHMAIR air chemistry code[2] with a simple hydrodynamic code and the external circuit equations. The predicted expansion was consistent with Schlieren and image converter camera photographs of the channel provided suitable initial conditions were chosen. Justifying these initial conditions, and showing that they are consistent with our measurements of the breakdown

Manuscript submitted March 8, 1982.

process, is the subject of ongoing research which in part depends on understanding the radiative processes occurring in and around the channel.

The simulation code followed electron, gas, and vibrational temperatures and various species densities as well as the radial expansion. The values taken as representative for the first current peak and used to estimate radiation losses are

$$
\begin{aligned}
T_e &= 20,000 \text{ K}, \\
N_e &= 1.6 \times 10^{18} \text{ cm}^{-3}, \\
N_n &= 6 \times 10^{16} \text{ cm}^{-3}, \\
\text{radius} &= .25 \text{ cm}
\end{aligned}
$$

where N_n is the neutral density. The dominance of NII and OII lines in the visible spectrum (Fig. 1) supports these estimates of the conditions.

During the next 29 μs the hot gas within the channel expands to reach pressure equilibrium with the atmosphere around it. At equilibrium the measured channel radius is $\sim$1.1 cm, and we estimate[1] that the equilibrium temperature is $T_e \sim T_g \sim 5000$ K. To determine an upper limit of the conductive cooling we assume a time average channel temperate of $\sim$7000 K and a radius of $\sim$1 cm.

III. CONDUCTIVE COOLING

Determining the conductive cooling of a hot body generally requires solving the heat conduction equation[4]

$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T \tag{1}$$

subject to initial and boundary conditions. In this equation α is the thermal diffusivity and is related to the thermal conductivity, k, the specific heat, c, and the density, ρ, of the conducting medium by the relation

$$\alpha = \frac{k}{\rho c}. \tag{2}$$

We assume that the hot air in the channel can transport heat quickly compared with the cold outside air We also assume that the heat loss from the channel will prove to be small. We may then consider

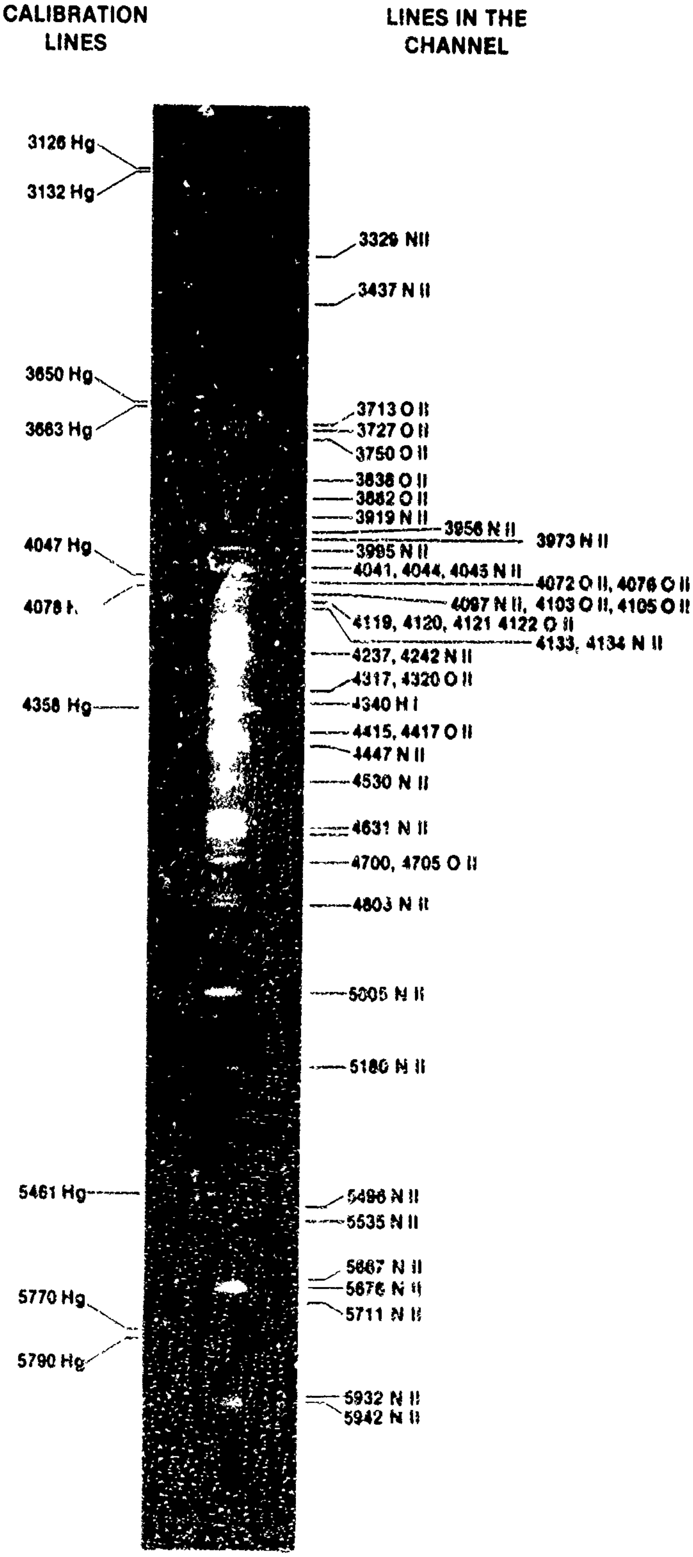

Fig 1 – The time integrated visible spectrum of the channel the channel to be a heat source maintained at a surface temperature of ~ 7000 K. It is not necessary to solve Eq. (1) in a cylindrical geometry because the heat from the channel will penetrate only a short distance in 30 μs. We may make a simple one dimensional argument for the heat loss rate per unit surface area.[4] If the heat flows outward from the channel to a characteristic depth δ, there exists a mean temperature gradient of T/δ (T is the channel temperature which is large compared to the ambient temperature). There exists a corresponding outward heat flow per unit surface area of kT/δ. The section of boundary layer in contact with the unit surface area has a mean temperature of $T/2$ and is therefore absorbing heat at a rate of $\rho c \left(\frac{T}{2} \right) \dot{\delta}$ where $\dot{\delta}$ is the rate of increase of the boundary layer thickness and the rate of increase of the volume of the boundary layer section. The characteristic value of $\dot{\delta}$ is δ / t. We may equate the rate at which energy leaves the channel surface and the rate of energy increase in the boundary layer

$$\frac{kT}{\delta} = \rho c \left(\frac{T}{2} \right) \frac{\delta}{t}. \tag{3}$$

The thickness of the boundary layer after time t is therefore given by

$$\delta^2 = 2 \alpha t. \tag{4}$$

[We note that δ in Eq. (4) is just a factor of $\sqrt{2}$ larger than the characteristic width of the boundary layer given by the exact solution of the diffusion equation; see for example, Carslaw and Jaeger, "Conduction of Heat in Solids," Clarendon Press, Oxford, 1959 p. 51.] Substituting the value of thermal diffusivity for ambient air, we obtain the boundary layer thickness at 30 μs,

$$\delta \approx 4 \times 10^{-5} \ m. \tag{5}$$

The mean heat flow per unit surface area is then

$$\frac{kT}{\delta} \sim 5 \times 10^6 \ \frac{\mathrm{J}}{\mathrm{sec\ m^2}}. \tag{6}$$

This flow rate, sustained for 30 μs over the surface area of a unit length of channel, gives a total energy loss of $\sim 9 \times 10^{-2}$ J/cm, and is negligible compared to the ~ 3 J/cm of energy in a unit length of channel.

IV. LOSSES DUE TO ELECTRON-ION BREMSSTRAHLUNG

A summary of the treatment presented in Zel'dovich and Raizer[5] for deriving the "integrated emission coefficient" for electron-ion bremsstrahlung from the "effective radiation" due to Coulomb scattering will be given (and will serve as a guide for the electron-neutral and bound-free calculations). The effective radiation is defined as the total energy emitted between frequencies ν and $\nu + d\nu$ when a unit electron flux passes by one ion (e.g. one electron of specified velocity and moving parallel to the z axis traverses each square cm of an x-y plane containing the ion). The effective radiation is given by

$$dq_\nu \simeq \frac{32\pi^2 Z^2 e^6}{3\sqrt{3}\, m^2 c^3 \mathrm{v}^2}\, d\nu \tag{7}$$

where Z is the ion charge, m the electron mass, and v the electron velocity. This approximation holds for electron velocities satisfying the quasi-classical condition

$$\frac{h\mathrm{v}}{2\pi Z e^2} << 1 \tag{8}$$

which is true in normal density air up to temperatures of $\sim 10^6$ K. To account for the actual number of electrons in the gas we assume they have a Maxwellian velocity distribution

$$f(\underline{\mathrm{v}})\, d^3\mathrm{v} = \left(\frac{m}{2\pi kT}\right)^{3/2} e^{-m\mathrm{v}^2/2kT}\, d^3\mathrm{v} \tag{9}$$

and that it is isotropic. Each unit volume then contains $N_e f(\underline{\mathrm{v}})\, d^3\mathrm{v}$ electrons moving in any given direction with velocity $\underline{\mathrm{v}}$. Thus there appears to be a uniform electron flux of $\mathrm{v} N_e f(\underline{\mathrm{v}})\, d^3\mathrm{v}$ in that direction. The total flux impinging on an ion is found by summing over all directions.

$$\begin{aligned} \int_{\text{all angles}} \mathrm{v} N_e\, f(\underline{\mathrm{v}})\, d^3\mathrm{v} &= \mathrm{v} N_e\, f(\underline{\mathrm{v}})\, 4\pi \mathrm{v}^2 d\mathrm{v} \\ &= \mathrm{v} N_e\, F(\mathrm{v})\, d\mathrm{v} \end{aligned} \tag{10}$$

where $F(\mathrm{v})$ is the Maxwellian speed distribution

$$F(\mathrm{v})\, d\mathrm{v} = 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} \mathrm{v}^2\, e^{-m\mathrm{v}^2/2kT}\, d\mathrm{v}. \tag{11}$$

Multiplying the effective radiation by this electron flux and by the number density of ions gives the energy emitted per second per unit volume between frequencies ν and $\nu + d\nu$ due to electrons with speeds between v and v + dv.

$$J_{\nu,\mathrm{v}}\, d\nu\, d\mathrm{v} = N^+ N_e\, \mathrm{v}\, F(\mathrm{v})\, d\mathrm{v}\, dq_\nu(\mathrm{v}). \tag{12}$$

The "spectral emission coefficient," J_ν, is defined as the energy emitted per second per unit volume between frequencies ν and $\nu + d\nu$ and is obtained by summing expression (16) over all electron speeds sufficient to create photons of energy $h\nu$

$$J_\nu d\nu = \int_{v_{min}}^{\infty} N^+ N_e \, vF(v) \, dq_\nu(v) \, dv. \tag{13}$$

The minimum speed is determined by

$$\frac{1}{2} m v_{min}^2 = h\nu. \tag{14}$$

Substituting from relations (7), (11) and (14) and integrating yields

$$J_\nu d\nu = \frac{32\pi}{3} \left(\frac{2\pi}{3kTm} \right)^{1/2} \frac{Z^2 e^6}{mc^3} N^+ N_e \, e^{-h\nu/kT} \, d\nu \tag{15}$$

The integrated emission coefficient, J_{ei}, is defined as the energy emitted per second per unit volume and is found by summing the spectral emission coefficient over all frequencies.

$$J_{ei} = \int_0^\infty J_\nu d\nu = \frac{32\pi}{3} \left(\frac{2\pi kT}{3m} \right)^{1/2} \frac{Z^2 e^6}{mc^3 h} N^+ N_e \frac{erg}{\mathrm{cm}^3 \ \mathrm{sec}}. \tag{16}$$

For the conditions in the channel the integrated emission coefficient equals $\sim 5.1 \times 10^4$ J/sec cm^3.

V. LOSSES DUE TO ELECTRON-NEUTRAL BREMSSTRAHLUNG

The integrated emission coefficient for electron-neutral bremsstrahlung may be found by starting with the effective radiation for electron-neutral collisions[5]

$$dq_\nu = \frac{8 \, e^2 v^2 \sigma_{ne}}{3c^3} \, d\nu \tag{17}$$

where σ_{ne} is the momentum transfer cross section. A Maxwellian distribution may again be assumed for evaluating the electron flux. We sum the radiation resulting from all electrons of speed exceeding v_{min} and multiply by the neutral density (collective effects are negligible for our conditions) to find the spectral emission coefficient

$$J_\nu d_\nu = \frac{32 \, \pi \, e^2 \, N_n N_e \, \sigma_{ne}}{3 \, c^3} \left(\frac{m}{2\pi kT} \right)^{3/2} \int_{v_{min}}^{\infty} v^5 \, e^{-mv^2/2kT} \, dv. \tag{18}$$

The evaluation of the integral is accomplished by repeated integration by parts

$$\int x^5 e^{-x^2} dx = -e^{-x^2} \left(\frac{x^4}{2} + x^2 + \frac{1}{2} \right) + C. \tag{19}$$

The spectral emission coefficient therefore becomes

$$J_\nu d\nu = \frac{32\pi\ e^2 N_n N_e\ \sigma_{ne}}{3\ c^3} \left\{\frac{2kT}{\pi m}\right\}^{3/2} \left\{\frac{1}{2}\left(\frac{h\nu}{kT}\right)^2 + \frac{h\nu}{kT} + \frac{1}{2}\right\} e^{-\frac{h\nu}{kT}} d\nu. \tag{20}$$

Integration over all frequencies gives the integrated emission coefficient

$$J_{en} = \frac{80\ e^2\ N_n N_e\ \sigma_{ne}}{3\ c^3\ h\ \sqrt{\pi}} \left\{\frac{2}{m}\right\}^{3/2} (kT)^{5/2}\ \frac{erg}{\text{sec cm}^3}. \tag{21}$$

Using a representative value[7] of σ_{ne} ($\sim 10^{-15}$ cm^2) we find the integrated emission coefficient to be $\sim 2.4 \times 10^2$ J/sec cm^3.

VI. LOSSES DUE TO FREE-BOUND TRANSITIONS

We will estimate the radiation losses from the recombination of electrons and ions by treating the bound states as hydrogen like. The energy levels are then given by

$$E_n = -\frac{2\pi^2\ me^4\ Z^2}{h^2 n^2}. \tag{22}$$

The classical formula for the effective radiation due to electron-ion bremsstrahlung (7) may be considered as describing a transition between hyperbolic electron orbits. The electron motion in the higher bound levels may also be treated as quasi-classical. Extending relation (7) to describe a transition between hyperbolic and eliptic orbits, leads to an estimate of the electron capture cross section into a given level.[8]

$$\sigma_{cn} = \frac{128}{3\sqrt{3}} \frac{\pi^2 Z^4 e^{10}}{mc^3 v^2 h^4 \nu\ n^3}. \tag{23}$$

The energy emitted per second, per unit volume from transitions by an electron of a particular speed into a given level is given by

$$J_{v,n} = (h\nu)\ (N^+\sigma_{cn})(vN_e\ F(v)\ dv). \tag{24}$$

Substituting for σ_{cn} yields

$$J_{v,n} = \frac{128\ \pi^4\ N^+ N_e\ Z^4\ e^{10}}{3\sqrt{3}\ m\ c^3 v\ h^3 n^3}\ F(v)\ dv. \tag{25}$$

The total energy emitted per second, per unit volume is found by summing over all speeds and all levels. (Extending (23) beyond quasi-classical limits can be justified by comparison to quantum mechanical calculations)

$$J = \frac{128\pi^4 N^+ N_e\, Z^4\, e^{10}}{3\sqrt{3}\, m\, c^3 h^3} \int_0^\infty \frac{F(v)\,dv}{v} \sum_{n=1}^{\infty} \frac{1}{n^3}. \tag{26}$$

Inter-particle interactions lower the continuum threshold leaving only a finite number of distinguishable levels. The summation therefore remains finite and we take it to be of order unity. Substituting for $F(v)$ and performing the integration yields the integrated emission coefficient for free-bound transitions

$$J_{fb} = \frac{512\,\pi^5\, N^+ N_e\, Z^4\, e^{10}\, kT}{3\sqrt{3}\, m^2\, c^3\, h^3} \left(\frac{m}{2\pi kT}\right)^{3/2} = \frac{2J_{el}\, E_{\text{ionization}}}{kT} \frac{erg}{\sec\ \text{cm}^3}. \tag{27}$$

Substitution of the channel values gives $\sim 8.6 \times 10^5$ J/sec cm^3.

VII. LOSSES DUE TO LINE RADIATION

Tabulated transition probabilities and multiplicities[9] allow us to calculate the line radiation for the dominant species NI, NII, OI, and OII. Initially we assume all lines are optically thin. We let α represent the quantum numbers of the upper level of a line and α' the quantum numbers of the lower level. The number density of the upper level is N_α, the spontaneous transition probability $A_{\alpha\alpha'}$, and the frequency of the line is $\nu_{\alpha\alpha'}$. The power emitted per unit volume in one line is then given by[10]

$$P_{\alpha\alpha'} = N_\alpha A_{\alpha\alpha'} h\nu_{\alpha\alpha'} \frac{erg}{\text{cm}^3\ \sec}. \tag{28}$$

The channel will be in a state of partial thermal equilibrium and the densities of various excited states of the ionic species will be determined by the total ion density N^+ and the electron temperature T_e[11]

$$N_\alpha = \frac{N^+ g_\alpha e^{-E_\alpha/kT_e}}{Z_+(T_e)}\ \text{cm}^{-3} \tag{29}$$

where g_α is the multiplicity of the level α, E_α is the excitation energy of this level and $Z_+(T_e)$ is the partition function of the ion as a function of temperature and is available in tabulated form.[12] The power emitted in a particular ion line is then

$$P_{\alpha\alpha'} = \frac{N^+ g_\alpha e^{-E_\alpha/kT_e}}{Z_+(T_e)} A_{\alpha\alpha'} h\nu_{\alpha\alpha'} \tag{30}$$

and the total power radiated by all lines from this ion species is given by

$$P = \frac{hN^+}{Z_+(T_e)} \sum_{\alpha,\alpha'} g_\alpha e^{-E_\alpha/kT_e} A_{\alpha\alpha'} \nu_{\alpha\alpha'} \frac{erg}{\mathrm{cm}^3\ \mathrm{sec}}. \tag{31}$$

A similar equation with N^0 replacing N^+ and $Z_0(T_e)$ replacing $Z_+(T_e)$ gives the total power radiated in the lines from the neutral atom.

The bulk of the emitted radiation is in the ultraviolet. Summing over the resonance series for NII and OII gives

$$P \sim 5 \times 10^6\ \mathrm{J/sec\ cm^3}$$

and over the same series for OI and NI

$$P \sim 1 \times 10^6\ \mathrm{J/sec\ cm^3}.$$

VIII. REABSORPTION OF RADIATION

Outside the channel the cold air will be quite transparent to the visible frequencies but essentially opaque below ~1800 Å due to the Schumann-Runge band system of the oxygen molecule,[13] ($l \sim 0.1$ mm for $\lambda \sim 1500$ Å). Thus if significant amounts of radiation are emitted from the hot channel they will be absorbed in the cold gas around the channel, so broadening the hot channel.

Within the channel the reverse processes of inverse electron-ion bremsstrahlung, inverse electron-neutral bremsstrahlung, and photo ionization will each contribute to the opacity of the channel and we may estimate the resulting frequency averaged absorption coefficient.

In an infinite medium in thermal equilibrium, there exits an isotropic total incident power of $4\sigma T^4$ where σ is the Stephan-Boltzman constant. In order to balance emission and absorption within the medium there must exist a frequency averaged absorption coefficient given by

$$K_1 \approx \frac{J_{ei} + J_{en} + J_{ib}}{4\sigma\ T^4}$$

The corresponding mean free path $l_1 = 1/K_1$ may be evaluated using our previous values for the integrated emission coefficients and the channel temperature. The resulting value of $l_1 \approx 4$ cm justifies the assumption that the channel is optically thin to the continuum.

Stark broadening parameters[14] indicate that the ultraviolet ion lines will be collisionly broadened to ~.05 Å and that they will have absorption lengths of $\sim 2 \times 10^{-4}$ mm within the channel. The visible lines will be Stark broadened to widths of ~4 Å corresponding to absorption lengths of ~1 cm. Visible emissions will therefore escape the channel. Experimental measurements of the visible line widths (Fig. 2) are in reasonable agreement, indicating that the electron density estimate is good.

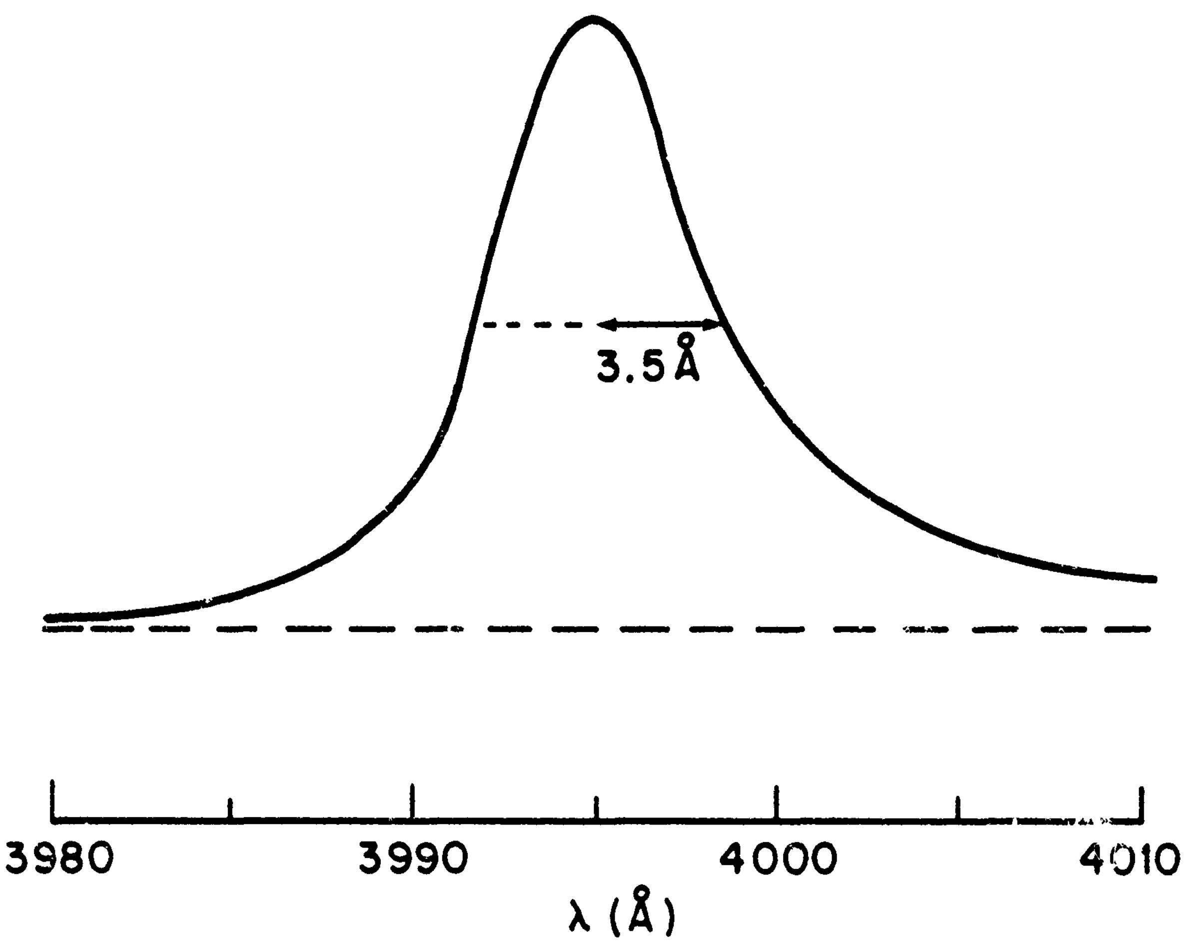

Fig 2 — The profile of the NII line at 3995 Å

IX. LOSSES DUE TO OPTICALLY THICK LINE RADIATION

We have seen that the u.v. lines will be optically thick. Their profiles, as seen at the channel surface, are broadened and saturated at the black body level. The power emitted per frequency interval and per unit surface area is[15]

$$I_\nu = S(\nu,T)\,\{1 - e^{-\tau}\} \qquad (32)$$

where τ is the optical depth of the emitting medium and $S(\nu,T)$ is the Planck function integrated over 2π steradians[16]

$$S(\nu,T) = \frac{2\pi h\nu^3}{c^2}\left\{e^{\frac{h\nu}{kT}} - 1\right\}^{-1}. \qquad (33)$$

The Planck function is approximately constant over the width of the line so the total power emitted per unit area in the line is

$$I \approx 2\Delta\nu\, S. \qquad (34)$$

We must evaluate the half width, $\Delta\nu$, of the profile represented by Eq. (32). The half intensity points occur where

$$\tau(\nu \pm \Delta\nu) = \ln 2. \qquad (35)$$

The optical depth is a function of the thickness of the plasma, l, the density of atoms in the lower state of the line, $N_{\alpha'}$, and the absorption cross section per atom $\sigma(\nu)$.

$$\tau(\nu) = \sigma(\nu)N_{\alpha'}l. \qquad (36)$$

The density, $N_{\alpha'}$, is given by Eq. (29) and the absorption cross section is approximately Lorentzian in shape with the width determined by the Stark broadening[17]

$$\sigma(\nu) = \frac{c^2}{8\pi^2\nu^2}\,\frac{g_{\alpha'}}{g_\alpha}\,A_{\alpha\alpha'}\,\frac{\Gamma}{4\pi}\left\{\frac{1}{(\Delta\nu)^2 + \left(\frac{\Gamma}{4\pi}\right)^2}\right\} \qquad (37)$$

where

$$\frac{\Gamma}{2\pi} = \text{full width at half maximum.}$$

The half width, $\Delta\nu$, of the optically thick line then follows from

$$\sigma(\nu \pm \Delta\nu) = \frac{\ln 2}{N_\alpha \cdot l} \tag{38}$$

where we have combined Eqs. (35), (36) and (37). We determine $\Delta\nu$ for each u.v. line by using Eq. (38), taking Γ from the Stark broadening data, and setting l equal to the column diameter ($l \approx .5$ cm). Using these values of $\Delta\nu$ we evaluate I for each line via Eq. (34). Summing over the NII and OII lines and multiplying by the surface area of a unit length of channel gives the total emission per unit length

$$\text{NII, OII} \qquad P \approx 1.5 \times 10^4 \text{ J/sec cm.}$$

The density of neutral atomic species is considerably less than the ion density; however, because the atomic levels are broadened more than the ionic levels, the atoms make a larger contribution to the emitted power,

$$\text{NI, OI} \qquad P \approx 5 \times 10^4 \text{ J/sec cm.}$$

X. CONCLUSIONS

For the conditions calculated as representing the electric discharge heated channel at the first current peak, the emissions from a unit length of channel ($r \sim .25$ cm) due to various radiative mechanisms are as follows;

Electron Ion Brem.	1×10^4 J/sec cm,
Electron Neutral Brem.	5×10^1 J/sec cm,
Recomb. Radiation	2×10^5 J/sec cm,
Line Radiation (thick)	6.5×10^4 J/sec cm.

These losses are small compared to the rate of energy deposition within the channel ($\sim 10^6$ J/sec cm) at that time. Heat conduction losses have also been seen to be small. Thus the assumption of adiabatic expansion is reasonable for an approximate treatment of the hydrodynamic expansion to pressure equilibrium. However of the four different processes of radiation emission, one, radiation in optically thick lines, is strongly temperature dependent. Therefore if, for instance, the same electrical energy were deposited in a channel of only one half the present diameter (0 25 cm instead of 0.5 cm) so that the temperature rose to $T_e \sim 80{,}000$ K, the energy radiated in optically thick lines would rise more than $\times 100$. Clearly such radiation output would exceed the energy input and the hot channel would not be able to sustain itself. At the same time, the emitted radiation would be absorbed in the cold air around the original channel in an annulus $\lesssim 1$ mm thick. Thus the channel would rapidly be broadened to a diameter of ~ 0.5 cm and the temperature reduced to a value at which radiation losses were no longer significant. In this way line radiation effectively limits the temperature to which air can be heated by an electric discharge to values of $\lesssim 20{,}000$ K since the electrical conductivity[18] varies only as $T^{3/2}$.

XI. ACKNOWLEDGMENTS

One of us (M. Raleigh) thanks Drs. H.L. Crannell and C.W. Werntz of the Catholic University of America who monitored the independent studies of which this material formed a part. We thank Dr. A.W. Ali both for his contributions to the development of the CHMAIR code and for many discussions of problems in air chemistry. We also acknowledge the interest taken in this topic by Dr. R.E. Pechacek of NRL.

XII. REFERENCES

1. M. Raleigh, J.R. Greig, R.E. Pechacek, and E. Laikin, NRL MR 4380 (1981).

2. R.F. Fernsler, A.W. Ali, J.R. Greig, and I.M. Vitkovitsky, NRL MR 4110 (1979).

3. M. Lampe, H. Harold Szu, and S. Kainer, NRL MR 4073, (1979). See also S. Kainer, M. Lampe, M. Raleigh, J.R. Greig, R.E. Pechacek, and R.F Fernsler, Bull. Am. Phys. Soc. *25*, 853 (1980).

4. E.U. Condon and H. Odishaw, "Handbook of Physics," 2nd ed., McGraw-Hill, New York, 1967.

5. Y B. Zel'dovich and Y.P. Raizer, "Physics of Shock Waves and High Temperature Hydrodynamic Phenomena," Academic Press, New York 1966, Vol. I, p. 258.

6. Ibid, p. 255.

7. L.G.H. Huxley and R.W. Crompton in "Atomic and Molecular Processes" edited by D.R. Bates, Academic Press, New York 1962, p. 363.

8. Y.B Zel'dovich and Y.P. Raizer, Op Cit., p. 263.

9. W.L. Wiese, M.W. Smith, and B.M. Glennon, "Atomic Transition Probabilities Volume I Hydrogen through Neon," National Bureau of Standards, Washington, 1966.

10. H.R. Griem, "Plasma Spectroscopy," McGraw-Hill, New York, 1964, p. 27.

11. Ibid, p. 134.

12. H.W. Drawin and P. Felenbok, "Data for Plasmas in Local Thermodynamic Equilibrium," Gauthier-Villars, Paris, 1965.

13. Y.B. Zel'dovich and Y.P. Raizer, Op. Cit., Vol. II, p. 604.

14. H.R. Griem, "Spectral Line Broadening by Plasmas," Academic Press, New York, p. 370, 1974.

15. H.R. Griem, "Plasma Spectroscopy," McGraw Hill, New York, 1964, p. 289.

16. Y.B. Zel'dovich and Y.P. Raizer, Op Cit, p. 117.

17. Ibid, p. 292.

18. L. Spitzer, "Physics of Fully Ionized Gases," 2nd rev. ed., Interscience, New York, 1962, p. 136.